### JUSTIFYING EMPIRICAL MACRO-ECONOMETRIC EVIDENCE IN PRACTICE

Neil R. Ericsson\*

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Abstract: Hendry (2011) outlines a coherent framework for generating, analyzing, and justifying empirical macro-econometric evidence. The current paper focuses on two key tools in that framework—automated model selection and impulse indicator saturation—and illustrates and generalizes those tools by re-analyzing the empirical model of seasonally unadjusted UK narrow money demand in Ericsson, Hendry, and Tran (1994). Both tools demonstrate the robustness of that model to a wide range of feasible alternatives. Those tools also yield statistical and economic improvements to that model and, in so doing, provide insights into the practical justification of empirical evidence in macro-economics.

*Keywords*: Autometrics, dynamic specification, impulse indicator saturation, model selection, money demand, United Kingdom.

JEL classifications: C52, E41.

# 1 Introduction

Economists are highly divided in their views on the role of empirical evidence in the profession. Hendry (2011) outlines a coherent framework for generating, analyzing, and justifying empirical macro-econometric evidence. The current paper discusses two key tools in that framework—automated model selection and impulse indicator saturation—and illustrates and generalizes those tools by re-analyzing the empirical model of seasonally unadjusted UK narrow money demand in Ericsson, Hendry, and Tran (1994). Both tools demonstrate the robustness of that model to a wide range of feasible alternatives. Those tools also yield statistical and economic improvements to that model and, in so doing, provide insights into the practical justification of empirical evidence in macro-economics.

<sup>\*</sup>Prepared for the online conference "Communications with Economists: Current and Future Trends" commemorating the 25th anniversary of the *Journal of Economic Surveys*. The author is a staff economist in the Division of International Finance, Board of Governors of the Federal Reserve System, Washington, DC 20551 USA, and a Research Professor of Economics, Department of Economics, The George Washington University, Washington, DC 20052 USA. He may be reached on the Internet at ericsson@frb.gov and ericsson@gwu.edu. The views in this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. The author is grateful to Jennifer Castle, Svetlana Chekmasova, Jurgen Doornik, David Hendry, Stedman Hood, Søren Johansen, and Bent Nielsen for helpful discussions and comments. All numerical results were obtained using PcGive Version 13.2 and Autometrics Version 1.5e in OxMetrics Version 6.20: see Doornik and Hendry (2009) and Doornik (2009).

This paper is organized as follows. Section 2 briefly describes the economic theory, data, and empirical model in Ericsson, Hendry, and Tran (1994); and it re-evaluates that model with automated model selection. Section 3 proposes extensions to impulse indicator saturation, and it employs impulse indicator saturation and some of those extensions to test for parameter constancy in Ericsson, Hendry, and Tran's (1994) model. Section 4 develops a new empirical model of UK narrow money demand in light of the methods and evidence in Sections 2 and 3. Section 5 offers a few general remarks on empirical evidence in macro-econometric modeling.

# 2 A Model of Seasonally Unadjusted UK Narrow Money Demand

Ericsson, Hendry, and Tran (1994) (denoted EHT below) design an empirical model of UK narrow money demand, using seasonally unadjusted data. EHT analyze the measure of nominal narrow money  $M_1$  (M), real total final expenditure (TFE) at 1985 prices (Y), the TFE deflator (P), and a net interest rate ( $R^{net}$ ) that aims to capture the opportunity cost of holding money. Money and expenditure are in £ millions, the deflator is unity for 1985, and the interest rate is in fractions. The data are quarterly, 1963Q1–1989Q2. Allowing for lags and transformations, estimation is over 1964Q3– 1989Q2, which is 100 observations (T = 100). For notational convenience, lowercase letters denote logs.

EHT test for and find a single cointegrating vector in a fifth-order vector autoregression of real money (m-p), real income (y), inflation  $(\Delta p)$ , and the net interest rate  $(R^{net})$ , including a constant term and seasonal dummies  $(Q_{1t}, Q_{2t}, Q_{3t})$ . Furthermore, the variables y,  $\Delta p$ , and  $R^{net}$  appear to be weakly exogenous for that cointegrating vector, which is interpretable as a money demand relationship. EHT thus turn to modeling an equilibrium correction model of real money, starting with a fifth-order autoregressive distributed lag (ADL) in m - p, y,  $\Delta p$ ,  $R^{net}$ , a constant term, and seasonal dummies. EHT manually simplify that ADL to the following specification.

$$\widehat{\Delta(m-p)_{t}} = - \underbrace{0.95}_{(0.12)} [\Delta_{3}(m-p)_{t-1}/3] - \underbrace{1.07}_{(0.15)} [(\Delta p_{t} + \Delta p_{t-4})/2] \\
+ \underbrace{0.16}_{(0.04)} \Delta^{2} y_{t-2} \\
- \underbrace{0.174}_{(0.013)} (m-p-y)_{t-1} - \underbrace{1.189}_{(0.093)} (\sum_{i=0}^{2} R_{t-i}^{net}/3) \\
+ \underbrace{0.038}_{(0.006)} - \underbrace{0.012}_{(0.005)} Q_{1t} + \underbrace{0.010}_{(0.005)} Q_{2t} + \underbrace{0.018}_{(0.007)} Q_{3t} \qquad (1)$$

$$\begin{split} T &= 100 \ [1964\text{Q3}-1989\text{Q2}] \quad \text{R}^2 = 0.84 \quad \hat{\sigma} = 1.348\% \quad Inn: F(24,67) = 0.70 \\ AR: F(5,86) &= 1.03 \quad ARCH: F(4,92) = 0.48 \quad Normality: \chi^2(2) = 5.63 \\ RESET: F(2,89) &= 0.81 \quad Hetero: F(13,86) = 1.26 \quad Form: F(23,76) = 0.98. \end{split}$$

Equation (1) appears well-specified on the statistics reported, including tests of parameter constancy by recursive least squares. See Doornik and Hendry (2009) and Doornik (2009) for a detailed description of the diagnostic statistics in equation (1).

Autometrics—in Doornik and Hendry's (2009) econometrics software package OxMetrics—implements automated model selection in a general-to-specific approach with diagnostic testing, multi-path search, and resolution of multiple terminal models through encompassing. Autometrics thus permits examining the possible path dependence of EHT's model selection: EHT's model in equation (1) arose from a manual general-to-specific model search that examined only a very limited number of potential simplification paths. Applying Autometrics at a 5% target size to a "naturally nesting" version of the unrestricted fifth-order ADL still obtains equation (1); see Ericsson (2010) on naturally nesting (and other types of) model representations. In obtaining equation (1), Autometrics considers  $2^{24}$  (over 16 million) different potential models.<sup>1</sup> Hence, equation (1) is clearly robust to a vast range of feasible alternatives, and so is robust to the model selection path. Section 3.2 re-examines equation (1) with impulse indicator saturation, and Section 4 develops a new empirical model on the existing evidence, with both sections employing automated model selection.

Above, and in Section 3.2, automated model selection is implemented as a *diag-nostic tool*, with the right-hand side variables from equation (1) forced (or "fixed") to enter the estimated equations in the model search. If those variables are needed in the final selected model, then forcing them to enter the model helps guide the search procedure. If those variables are not needed, then that should be apparent in the final model from the insignificance of their coefficients. In Section 4, automated model selection is implemented as a *model design tool*, albeit with the intercept, seasonal dummies, and long-run economic variables being fixed in the search process.

# 3 Impulse Indicator Saturation

Impulse indicator saturation (IIS) provides a general procedure for analyzing a model's constancy. Specifically, IIS is a generic test for an unknown number of breaks, occurring at unknown times, with unknown duration and magnitude, anywhere in the sample. Hendry (1999) proposes IIS as a procedure for testing parameter constancy; see Hendry, Johansen, and Santos (2008), Johansen and Nielsen (2009), and Hendry and Santos (2010) for further discussion and recent developments.

Many existing procedures can be interpreted as "special cases" of IIS in that they represent particular algorithmic implementations of IIS. Such special cases include recursive estimation, rolling regression, the Chow (1960) predictive failure statistic (including the 1-step, breakpoint, and forecast versions implemented in OxMetrics), the Andrews (1993) unknown breakpoint test, the Bai and Perron (1998) multiple breakpoint test, intercept correction (in forecasting), and robust estimation. IIS thus provides a natural conceptual framework for analyzing a model's constancy.

<sup>&</sup>lt;sup>1</sup>Although Autometrics considers  $2^{24}$  distinct potential models, it actually estimates only 27 of those models in this instance, owing to the sophisticated nature of Autometrics's search algorithm; see Doornik (2009) for details.

Algorithmically, IIS also solves the problem of having more potential regressors than observations by testing and selecting over blocks of variables. That approach aids interpretation of dynamic factor analysis, principal components, and similar procedures; and it leads to additional tests of parameter constancy. Section 3.1 thus re-interprets some existing tests as IIS-type tests, and it proposes some extensions to IIS. To illustrate, Section 3.2 uses IIS and some of those extensions to test for parameter constancy in EHT's model.

#### 3.1 Extensions

IIS is a powerful empirical tool for evaluating and improving existing empirical models. IIS also provides a conceptual framework for interpreting existing tests that leads to extensions of IIS itself.

Table 1 summarizes IIS, some other existing tests, and some extensions of IIS, all in terms of the variables involved. A few remarks may be helpful for interpreting the entries in Table 1. Throughout, T is the sample size, t is the index for time, i is the index for indicators, k is the index for economic variables (which are denoted  $x_{kt}$ ), and K is the total number of potential regressors considered.

- 1a. Impulse indicator saturation. This is the standard IIS procedure proposed by Hendry (1999), with selection among the T zero-one impulse indicators  $\{I_{it}\}$ .
- 1b. Super saturation. In addition to searching across  $\{I_{it}\}$ , super saturation searches across all possible one-off step functions  $\{S_{it}\}$ . Step functions are of economic interest because they may capture permanent or long-lasting changes in regime that are otherwise not incorporated into an empirical model. Statistically and numerically, a step function is a parsimonious representation of a sequential subset of impulse indicators that have equal coefficients.
- 1c. Super-duper saturation. Partial sums of the partial sums of impulse indicators may also be of economic interest, as those double-sums are broken linear trends  $\{T_{it}\}$ . Super-duper saturation searches across  $\{I_{it}, S_{it}, T_{it}\}$ . Obvious extensions are broken quadratic trends, broken cubic trends, and so forth.
- 1d. Sequential pairwise impulse indicator saturation. Extensions 1b and 1c are based on partial sums of the impulse indicators and step functions over the remaining sample, i.e., for all  $i \ge t$ . Partial sums also can be constructed over fixed-length windows of impulse indicators. The simplest case is sequential pairwise IIS, in which sequential pairs of impulse indicators are added together, i.e.,  $P_{it} = I_{it} + I_{i+1,t}$ . Pairs (or triplets, or quadruplets, etc.) may parsimoniously capture effects that are persistent but not permanent. Nonsequential groupwise IIS is also an option, with some nonsequential groups being of particular interest, such as groups at a seasonal frequency.
- 1e. **Zero-sum pairwise IIS**. Differences of impulse indicators may capture "zerosum" effects, with  $Z_{it} = \Delta I_{it}$ , and leading to zero-sum pairwise IIS. See Hendry (1974) and Campos and Ericsson (1999) for empirical examples.

Type	Name	Description	Variables	Definitions
1a	Impulse indicator saturation	zero-one dummies	$\{I_{it}\}$	$I_{it} = 1$ for $t = i$ , zero otherwise
1b	Super saturation	step functions	$\{I_{it}, S_{it}\}$	$S_{it} = 1$ for $t \ge i$ , zero otherwise
1c	Super- duper saturation	broken linear trends	$\{I_{it}, S_{it}, T_{it}\}$	$T_{it} = t - i + 1$ for $t \ge i$ , zero otherwise
1d	Sequential pairwise IIS	zero-one-one dummies	$\{P_{it}\}$	$P_{it} = 1$ for $t = i, i + 1;$ zero otherwise
1e	Zero-sum pairwise IIS	"plus-one"– "minus-one" dummies	$\{Z_{it}\}$	$Z_{it} = +1 \text{ for } t = i,$ $Z_{it} = -1 \text{ for } t = i + 1,$ zero otherwise
2	Many many variables	more variables than observations	$\{x_{kt}\}$	$x_{kt} = \sum_{i=1}^{T} x_{kt} I_{it}$
3	Factors	factors, principal components	$\{f_{jt}\}$	$f_{jt} = \sum_{\forall k} w_{jk} x_{kt}$
4	Multiplicative indicator saturation	partial series	$\{x_{kt}^{(i)}, \forall i, k\}$	$\begin{aligned} x_{kt}^{(i)} &= 0 \text{ for } t < i, \\ x_{kt}^{(i)} &= x_{kt} \text{ for } t \ge i \end{aligned}$

Table 1: Impulse indicator saturation and some extensions, as characterized by the variables involved.

2. Many many variables. IIS provides a solution for dealing with more potential variables than observations, i.e., K > T. In the same spirit, block searches can be applied to a set of economic variables for which there are more variables than observations. Additionally, every economic series is interpretable as the weighted sum of the impulse indicators  $\{I_{it}\}$ , where the weight on each impulse indicator is the value of the economic series for the observation corresponding to the impulse indicator. Block searches across many many variables are thus interpretable as searches across particular, economically interesting combinations of impulse indicators. Many models involve (say) K data aggregation assumptions with K > T in practice; those assumptions can now be tested. See Ericsson (2011) for an example of testing such aggregation assumptions in a global vector autoregression.

- 3. Factors. Factors and principal components are weighted sums of economic variables. So, from #2 above, they are weighted sums of the impulse indicators. See Stock and Watson (2002, 2005), Bernanke, Boivin, and Eliasz (2005), and Castle, Clements, and Hendry (2011) for discussions.
- 4. Multiplicative indicator saturation. If a model's coefficient on  $x_{kt}$  is suspected to have changed at a particular date i, a natural way to capture that change is by including  $S_{it}x_{kt}$  in the model, in addition to  $x_{kt}$ , with the coefficient on  $S_{it}x_{kt}$  picking up the incremental change in the original coefficient on  $x_{kt}$ . If the break-point i is itself unknown, block searches with more potential variables than observations permit considering  $S_{it}x_{kt}$  for all break-point dates i and variables k. This approach precisely nests the Andrews (1993) unknown breakpoint test and the Bai and Perron (1998) multiple breakpoint test, aside from directly allowing the error variance to alter. (IIS does allow the error variance to alter, but IIS is not very parsimonious in the way that it does so.)

While Table 1 details several interesting extensions of IIS, Table 1 is by no means an exhaustive list of extensions to IIS. Also, the choice of IIS-type procedure may itself be a combination of the entries in Table 1; and that choice may affect the power of the procedure to detect specific alternatives. For instance, the twenty-five impulse indicators  $\{I_{it}, i = 76, ..., 100\}$  are not a particularly parsimonious way of expressing a step shift that occurs three-quarters of the way through a sample of 100 observations, whereas the single step dummy  $S_{76,t}$  is.

### 3.2 An Application

This subsection applies IIS-type procedures to equation (1) using Autometrics. These procedures detect anomalies in 1969Q1 and 1969Q2, where those anomalies are of the same sign and virtually equal magnitude. A sequential pairwise impulse indicator for those two quarters provides a concise summary. Testing is conducted at an approximate 1/K target size; see Doornik (2009) and Johansen and Nielsen (2009).

The results with IIS-type procedures can be summarized as follows. IIS at a 1% target level (K = 100) detects two impulse indicators—one for 1969Q1, and one for 1969Q2—each with a coefficient of about -0.04, representing a growth rate of real money at about 4% per quarter lower than anticipated by the model. Super saturation (K = 200) does not detect any indicators at the 0.5% target level, but it does detect step dummies for 1969Q1 and 1969Q3 at the 1% target level, where the step dummies' coefficients are of opposite sign and virtually identical magnitude. Sequential pairwise IIS at the 0.33% target size (K = 300) detects a single paired impulse indicator (denoted  $P_t^{1969q1q2}$ ) that is unity for 1969Q1 and 1969Q2, thus providing a convenient representation of the results from IIS and super saturation. Multiplicative IS at the 0.1% target size (K = 1100) likewise obtains that single paired impulse indicator.

No obvious changes in data measurement appear to have occurred in 1969Q1 and 1969Q2; see Clews, Healey, Hoggarth, and Mann (1990) and Topping and Bishop

(1989). So, it remains to be determined what events induced a temporary decline in predicted real money in the first half of 1969, above and beyond the determinants already in equation (1).

### 4 Model Redesign with Autometrics and IIS

EHT (p. 207) note that equation (1) may be rewritten, re-interpreted, and (possibly) further simplified by transforming the dependent variable from the growth rate in real money (i.e.,  $\Delta(m-p)_t$ ) to the growth rate in nominal money (i.e.,  $\Delta m_t$ ). This section examines the implications of that transformation, using Autometrics to re-simplify the fifth-order ADL described in Section 2.

That ADL can be represented equivalently as an equation with the growth rate of nominal money as the dependent variable, even though the ADL determines the log-level of real money. Autometrics with multiplicative impulse saturation at a 0.1% target size (K = 1025) obtains the following simplification from that transformed ADL.

$$\widehat{\Delta m_t} = \left(\frac{\Delta_4 p_t}{4} - \frac{\Delta_3 m_{t-1}}{3}\right) + \underbrace{0.15}_{(0.04)} \Delta^2 (y+p)_{t-2} - \underbrace{0.042}_{(0.009)} P_t^{1969q1q2} \\ - \underbrace{0.161}_{(0.008)} (m-p-y)_{t-1} - \underbrace{1.171}_{(0.048)} (\sum_{i=0}^2 R_{t-i}^{net}/3) \\ + \underbrace{0.044}_{(0.005)} - \underbrace{0.011}_{(0.004)} Q_{1t} + \underbrace{0.009}_{(0.004)} Q_{2t} + \underbrace{0.015}_{(0.006)} Q_{3t}$$
(2)

$$\begin{split} T &= 100 \ [1964\text{Q3}-1989\text{Q2}] \quad \text{R}^2 = 0.82 \quad \hat{\sigma} = 1.23509\% \\ AR: \ F(5,87) &= 1.60 \quad ARCH: \ F(4,92) = 1.89 \quad Normality: \ \chi^2(2) = 1.56 \\ RESET: \ F(2,90) &= 1.33 \quad Hetero: \ F(13,86) = 2.09^* \quad Form: \ F(26,73) = 1.46. \end{split}$$

Equation (2) has a straightforward economic interpretation in line with standard Ss-type inventory models; see especially Miller and Orr (1966), Akerlof (1973, 1979), Akerlof and Milbourne (1980), Milbourne (1983), and Smith (1986). Nominal money growth equals the excess of annual inflation over nominal money growth in the past year, with additional adjustments from lagged money disequilibria, seasonality, and the acceleration of nominal income. Solved lag coefficients from equation (2) are very similar to those from equation (1). The dynamics of equation (2) is closely linked to the dynamic economic theory on which it is based. Specifically, the short-run dynamics of equation (2) is consistent with an Ss-type inventory model that has nominal short-run bounds, but where those bounds adjust to long-term effects.

Statistically, equation (2) is well-specified, apart from rejection by one test of heteroscedasticity. That rejection appears to arise from the inclusion of the sequential paired impulse indicator  $P_t^{1969q1q2}$ . The residual standard error in equation (2) is approximately 10% less than that in equation (1), primarily indicative of the statistical benefits of filtering the observations for 1969Q1–1969Q2.

Computer-automated model selection with Autometrics demonstrates the robustness of Ericsson, Hendry, and Tran's (1994) final equilibrium correction model to a wide range of feasible alternatives. Computer-automated model selection also improves on that model by using multi-path searches with IIS-type procedures. Those searches would be tedious and prohibitively time-consuming with standard econometric packages.

### 5 Remarks

Hendry (2011) sketches out how recent econometric tools and software developments have made empirical modeling an exciting and vibrant enterprise in economics. Additional results in the statistical theory of model selection—and corresponding software implementation—will no doubt further aid this endeavor. The current paper serves to exemplify and illustrate some aspects of how such empirical modeling might proceed.

# References

- Akerlof, G. A. (1973) "The Demand for Money: A General-equilibrium Inventorytheoretic Approach", *Review of Economic Studies*, 40, 1, 115–130.
- Akerlof, G. A. (1979) "Irving Fisher on His Head: The Consequences of Constant Threshold-target Monitoring of Money Holdings", Quarterly Journal of Economics, 93, 2, 169–187.
- Akerlof, G. A., and R. D. Milbourne (1980) "Irving Fisher on His Head II: The Consequences of the Timing of Payments for the Demand for Money", *Quarterly Journal of Economics*, 95, 1, 145–157.
- Andrews, D. W. K. (1993) "Tests for Parameter Instability and Structural Change with Unknown Change Point", *Econometrica*, 61, 4, 821–856.
- Bai, J., and P. Perron (1998) "Estimating and Testing Linear Models with Multiple Structural Changes", *Econometrica*, 66, 1, 47–78.
- Bernanke, B. S., J. Boivin, and P. Eliasz (2005) "Measuring the Effects of Monetary Policy: A Factor-augmented Vector Autoregressive (FAVAR) Approach", *Quarterly Journal of Economics*, 100, 1, 387–422.
- Campos, J., and N. R. Ericsson (1999) "Constructive Data Mining: Modeling Consumers' Expenditure in Venezuela", *Econometrics Journal*, 2, 2, 226–240.
- Castle, J. L., M. P. Clements, and D. F. Hendry (2011) "Forecasting by Factors, by Variables, by Both, or Neither?", mimeo, Institute for New Economic Thinking at the Oxford Martin School, University of Oxford, Oxford, June (paper presented at the Econometric Society European Meetings, Oslo, Norway, August, 2011).
- Chow, G. C. (1960) "Tests of Equality Between Sets of Coefficients in Two Linear Regressions", *Econometrica*, 28, 3, 591–605.

- Clews, R. D., J. E. C. Healey, G. Hoggarth, and C. R. Mann (1990) "Monetary Aggregates in a Changing Environment: A Statistical Discussion Paper", Bank of England Discussion Paper No. 47, Bank of England, London, March.
- Doornik, J. A. (2009) "Autometrics", Chapter 4 in J. L. Castle and N. Shephard (eds.) The Methodology and Practice of Econometrics: A Festschrift in Honour of David F. Hendry, Oxford University Press, Oxford, 88–121.
- Doornik, J. A., and D. F. Hendry (2009) *PcGive 13*, Timberlake Consultants Press, London (3 volumes).
- Ericsson, N. R. (2010) "Empirical Model Selection: Friedman and Schwartz Revisited", mimeo, Department of Economics, The George Washington University, Washington, D.C., March (paper presented at the 8th OxMetrics User Conference, Washington, DC, March 18–19, 2010).
- Ericsson, N. R. (2011) "Improving Global Vector Autoregressions", International Finance Discussion Paper, Board of Governors of the Federal Reserve System, Washington, D.C., in preparation.
- Ericsson, N. R., D. F. Hendry, and H.-A. Tran (1994) "Cointegration, Seasonality, Encompassing, and the Demand for Money in the United Kingdom", Chapter 7 in C. P. Hargreaves (ed.) Nonstationary Time Series Analysis and Cointegration, Oxford University Press, Oxford, 179–224.
- Hendry, D. F. (1974) "Stochastic Specification in an Aggregate Demand Model of the United Kingdom", *Econometrica*, 42, 3, 559–578.
- Hendry, D. F. (1999) "An Econometric Analysis of US Food Expenditure, 1931– 1989", Chapter 17 in J. R. Magnus and M. S. Morgan (eds.) Methodology and Tacit Knowledge: Two Experiments in Econometrics, John Wiley and Sons, Chichester, 341–361.
- Hendry, D. F. (2011) "Justifying Empirical Macro-econometric Evidence", Journal of Economic Surveys, Online 25th Anniversary Conference, November.
- Hendry, D. F., S. Johansen, and C. Santos (2008) "Automatic Selection of Indicators in a Fully Saturated Regression", *Computational Statistics*, 23, 2, 317–335, 337– 339.
- Hendry, D. F., and C. Santos (2010) "An Automatic Test of Super Exogeneity", Chapter 12 in T. Bollerslev, J. R. Russell, and M. W. Watson (eds.) Volatility and Time Series Econometrics: Essays in Honor of Robert F. Engle, Oxford University Press, Oxford, 164–193.
- Johansen, S., and B. Nielsen (2009) "An Analysis of the Indicator Saturation Estimator as a Robust Regression Estimator", Chapter 1 in J. L. Castle and N. Shephard (eds.) The Methodology and Practice of Econometrics: A Festschrift in Honour of David F. Hendry, Oxford University Press, Oxford, 1–36.
- Milbourne, R. (1983) "Optimal Money Holding Under Uncertainty", International Economic Review, 24, 3, 685–698.

- Miller, M. H., and D. Orr (1966) "A Model of the Demand for Money by Firms", *Quarterly Journal of Economics*, 80, 3, 413–435.
- Smith, G. W. (1986) "A Dynamic Baumol-Tobin Model of Money Demand", Review of Economic Studies, 53, 3, 465–469.
- Stock, J. H., and M. W. Watson (2002) "Macroeconomic Forecasting Using Diffusion Indexes", Journal of Business and Economic Statistics, 20, 2, 147–162.
- Stock, J. H., and M. W. Watson (2005) "Implications of Dynamic Factor Models for VAR Analysis", NBER Working Paper No. 11467, National Bureau of Economic Research, Cambridge, Massachusetts.
- Topping, S. L., and S. L. Bishop (1989) "Breaks in Monetary Series", Discussion Paper No. 23, Technical Series, Bank of England, London, February.